كلية التربية
المجلـــة العلمـــية

A Statistical Approach to Multi-Examinations

By

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دراسة إحصائية في نظام التعليم متعدد – الامتحانات

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ملخص البحث :-

يوضح البحث طريقة تطبيق طريقة احصائية لمعالجة نتائج الامتحانات المتعددة للمقرر الواحد. يقوم هذا عن طريق تقديم فضاء ثنائي الأبعاد و خط المتوقع . يساعد هذا الفضاء الإحصائي على الحكم على نوعية الامتحانات والتمييز بينها. و قد تم التوسع بالفكرة لتشمل امتحانات ذوات قيم تقدير مختلفة. كذلك يقدم البحث طريقة إحصائية لمعالجة حالات عدم توازن النتائج . يصبح تقييم الطالب بهذه الطريقة أكثر وضوحا و أكثر عدلا في المادة الواحدة أو المواد المختلفة. يمكن تطبيق الفكرة في نظامي التعليم ذو الامتحان الواحد و متعدد الامتحانات.

ABSTRACT

The study introduces a statistical treating of two-related - examinations to get a better understanding for the educational process. We introduce the hypothesis of an expectancy line in two dimensional space as a guide to the analysis. Such space shows an overall inter-and intra-comparison between examinations. Also, the technique of balancing the final result about this line is discussed.

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INTRODUCTION

To a student an examination is an event in his career to check on his educational progress. On the other hand, to a professor, an examination is an unavoidable procedure to evaluate the progress of the student in the subject. Basically, an examination is a classifying and a selecting process. However the results of an examination (i.e. the grades obtained by the students) are affected to different degrees by many different factors. For a single examination, one would study the grades distribution. It is not uncommon to find that the distribution is pedagogically unsatisfactory; e.g. The grades may be found to be biased to one side or another. The common and fair technique is to shift the (passing) average of the grades by some specific amount by adding the same amount to each grade. This procedure will not charge the frequency distribution. It only shifts the average. This is in accordance with the equation [1,2].

$$\frac{\sum (x_i \pm a) f_i}{\sum f_i} = \langle x \rangle \pm a \qquad (1)$$

The question arises: What is one to do with more-than-one related examinations? To give an example; the final grade of the student in applied sciences faculties is the sum of oral, experimental, and one or more mid-term beside the final written examinations. Another example, in the literature branche, one has to add mid–terms to the final examination. Multi-examinations seem to be now the current policy.

We just add the different results to get the final grade and then deal statistically with this final one. This is simple enough. This boils down to making all these different grades as parts of one examination. But, by smearing out the fine details the pedagogical aim of the multi-examinations is lost. Consequently, one has to treat the situation via multi-variables Statistics.

The present study is a Statistical Treatment of related examinations' grades of any group of students under the following situations: (i) Multiple examinations in the same subject under the same professor. (ii) Multiple examinations in different subjects under the same professor. (iii) Multiple examinations in the same subject under the different professors. (iv) ultiple examinations in different subjects under different professors.

The study is a small contribution we hope it would be beneficial to progress of the pedagogical field. Besides, it may be helpful to the staff in evaluating their performance. The approach is applied here to some Mathematics and Physics examinations' grades. The data are hypothetical ones but based on real cases from different universities in three different countries (Egypt, Iraq, and K.S.A.). The study is done with the Sample Theory in mind [1]. One has to remember that the grades of an examination are an example and consequently the treatment is justified. The treatment is based on Two Random Variables Graphic Technique used in experimental physics. The treatment gives an overall and detailed picture and helps the teaching staff in their decision making.

We take the reader step by step. The study is organized in three sections. In the first section, the Expectancy Hypothesis is introduced by dealing with two examination grades of equal weights. In the second section the idea is generalized to the results to two examinations of different grading weights. In the third section, examinations final grades are dealt with statistically to balance the examinations grades about the expectancy line.

Not to interrupt the flow of the arguments, remarks and explanations – indicated by superscripts - are added at the end of the article.

Expectancy Hypothesis

Let us introduce here the Expectancy Hypothesis defined by the following. Under the Expectancy hypothesis, "good students are expected to get always high grades in every examination and bad students will get always low marks in every one". The hypothesis is not a deterministic – exact - one but is subjected to wide probabilistic deviations – i.e. not only scatter but Biasing as well. To explain, let us take the grades of two examinations of the same group of students onto a two-dimensional space [x, y diagram] where each axis represents the grades of one of the examinations. According to the expectancy hypothesis one expects that the data clusters around the Expectancy Line – the 45o degree angle line or more precisely the line represented by the mathematical equation (y = x). Figure (1), shows such a presentation (1). The figure shows a random

distribution with some clustering around the expectancy line. One notices that though the frequency histograms of both data are nearly similar, there is no statistical correlation between the two results (2). The data does not follow closely the expectancy hypothesis. This is understandable if one is to remember that the grades are affected by many factors. Members of sub-groups in each data are interchangeable. (I.e. we have a regrouping process). Clearly, the grades of an examination process are highly random in nature!!! A regression analysis for such a random distribution is a waste of time [2]. Also, a study of the distributions' moments is not fruitful to our aim[1].

The data in Fig. (1) is scattered nearly equally on both sides of the consistency line. Unfortunately this is uncommon. One usually gets cases as in Fig. (2). The diagram is an example of biased results. Of course the different distributions which can be obtained are limitless. The density of the points if higher – or lower – on the upper side of the consistency line means that the students made better grades in the second – or the first – examination than the other one. We shall take care of such a problem in the third section.

Let us now divide the 2-D space in Fig.(1) into eight octants as indicated in the figure. This is done by taking the (50%, 50%) point - the (0.5, 0.5) point for a normalized scale - on the diagram as the hob of the octants (3). The octants are equal in area. The line orthogonal to the expectancy line (i.e. perpendicular to it) represents the "the total 50% passing grade" line. The diagram by this technique is very

helpful to the teaching Staff. From the diagram we propose to classify the students into four groups:

1: Those students who improved themselves on the second exam.

2: Those students who got 50% or more in the first.

3: Those students who got 50% or more in the second.

4: Those students who got 50% or more for the total grade.

The relation between these groups and the different octants are shown in the following table with recommendation in the last column.

octants	1	2	3	4	Recommendation
I	+	-	+	-	Encouragement
II	+	-	+	+	Not bad
III	+	+	+	+	Both are
IV	-	+	+	+	The best
V	-	+	-	+	Encouragement
VI	-	+	-	-	Need studying more
VII	-	-	-	-	Hopeless and
VIII	+	-	-	-	Or change specialty

Weight of the Examination

In the previous treatment the examinations are treated on equal footing. But let us suppose that the professor of the same course divides the course into three parts for example. He gives an examination at the end of the first-third, another one at the end of

the second-third, and the final examination at the end of the course. The first examination would cover the first-third of the course. The second examination may cover the second-third, or cover both the first and the second-third of the course. In the first case; the two examinations may be considered equal to some extent. In the second case; they may not be considered equal depending on the subject of the course. The same idea is applied to the final examination which can cover either: i. the last-third of the course, ii. the last two-thirds, or iii. the whole course. The grades of the examinations as a part of the whole are the decisive factor. This depends on the philosophy behind the examination itself.

Figure (3) shows the treatment of such an example, the first exam has a 20 points grade while the second is graded from 50 points.. The expectancy line is now represented by the equation [y = (5/2)x]. The set of passing lines defined by [y +x=Constant] divide the students to the known different classes according to grade percentages [A ≥ 90% , B≥ 80% , C ≥ 70% ...]. The octants can then be drawn as explained in the first section. They are not equal. So in general we have: The Expectancyy Line is represented by the line y = m x, where m is equal to the ratio of the two grades of the two examinations.

Data Treatment

In the previous two sections in dealing with the expectancy hypothesis, it is assumed implicitly that the examinations themselves are on the same level. Nobody can guaranty this. In this section a statistical approach is proposed to such a problem.

Let us take the data shown in Fig. 4. One notices that the second examination is "difficult" than the first examination. The data is biased and denser under the expectancy line. This is reflected onto the overall result of the two examinations. One needs to balance the two examinations to balance the final result.

To deal with final result's distribution as mentioned in the first section in connection with Eq.(1) , the technique here is altered to add a specific amount to each grade depending on its value, under the condition that the average of the result is raised by the same additional value as in Eq. (1). This is in accordance with the equation

$$\frac{\sum(x_i + bx_i) f_i}{\sum f_i} = < x > + b < x > \qquad \text{..................... (2)}$$

And if one takes the condition that

$$a = b < x > \qquad \text{..................... (3)}$$

One gets the same average as above. This will result in the rotation of the data anti-clockwise direction. The distribution of the grades will change a bit. But, one gets a better balance about the expectancy line. This is shown clearly in Fig.(5) if one is to compare it with Fig.(4). The change in the overall result is not worth mentioning (4). The same idea, one can also apply to the data in Fig. (2) or Fig. (3).

DISCUSSION AND CONCLUSION

The study introduces a new statistical technique to cope with related examinations more fairly than the simple treatments applied nowadays in our universities. It aids the teaching staff in their critical endeavors. In our opinion, it is more beneficial to the students as well(5).

The data used in the study are those from the science branch in the university - (understanding and memorizing). It can also be applied to the literature branch - (memorizing is the important part).

To deal with more than two examinations (3 or 4 e.g.) is really a challenge we shall try next.

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References

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Notes

1. The net results shown in the diagram are calculated along the consistency line and between lines parallel to the 50% line.
2. Some times, and not always the frequency distribution of the total results, reflect the frequency distribution of the first and or the second exam.
3. Some faculties in the universities take the 60% mark as the "passing grade". Consequently, the point (0.6, 0.6) will do as the hob of the octants. In this case, the octants will not be equal.
4. In Fig.(4), the mean value <x> of the first examination is about 4.7, while its value is about 3.6 for the second examination. Consequently; the value of "a" in equation (1) was taken equal to one and the value of "b" was taken equal to about 0.33 with the result as in Fig.(5).
5. One is assuming that the other factors: the different lecturers, different examinations .... etc, have negligible effect.

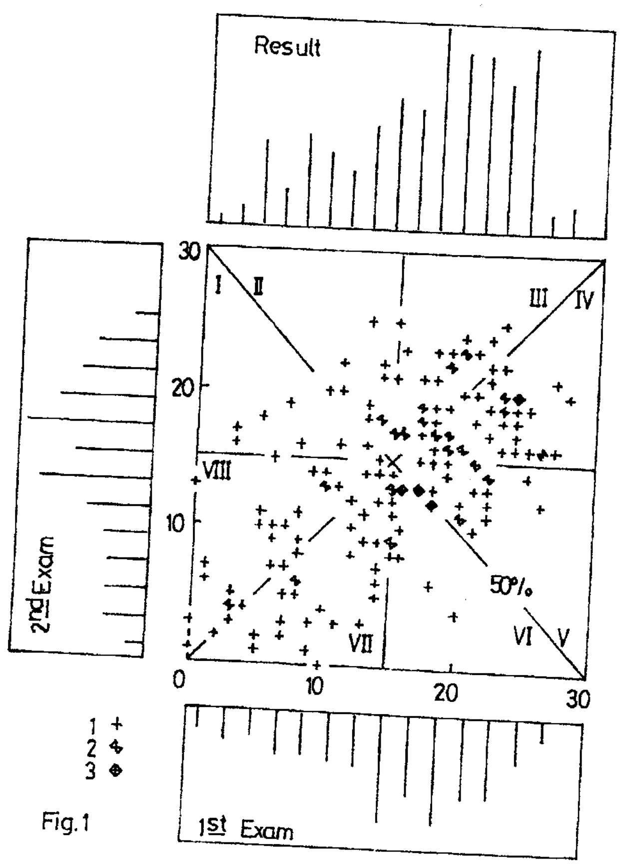
Result
30
20
10
0
10
20
30
I
II
III
IV
V
VI
VII
VIII
50%
2nd Exam
1st Exam
1 +
2
3

Fig.1

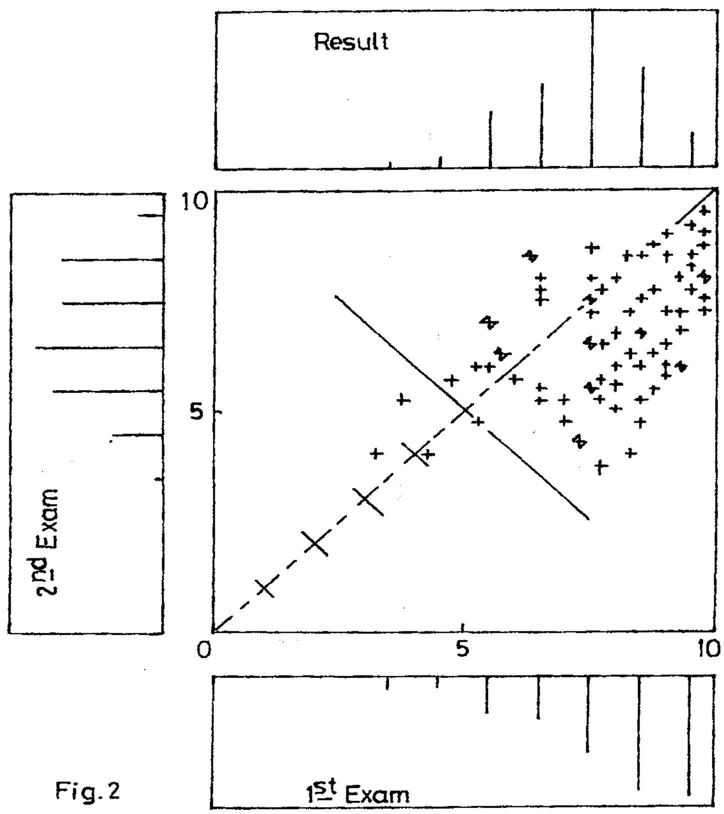

Fig. 2

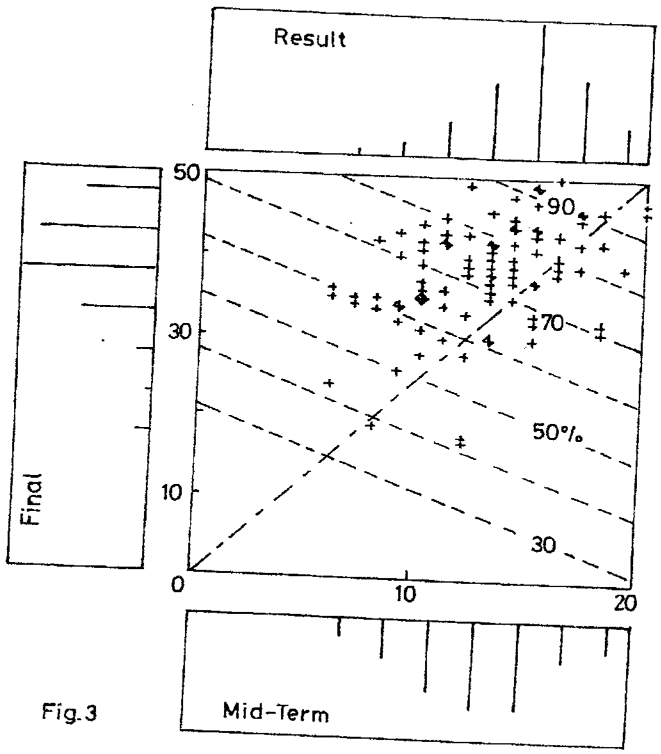

Fig.3

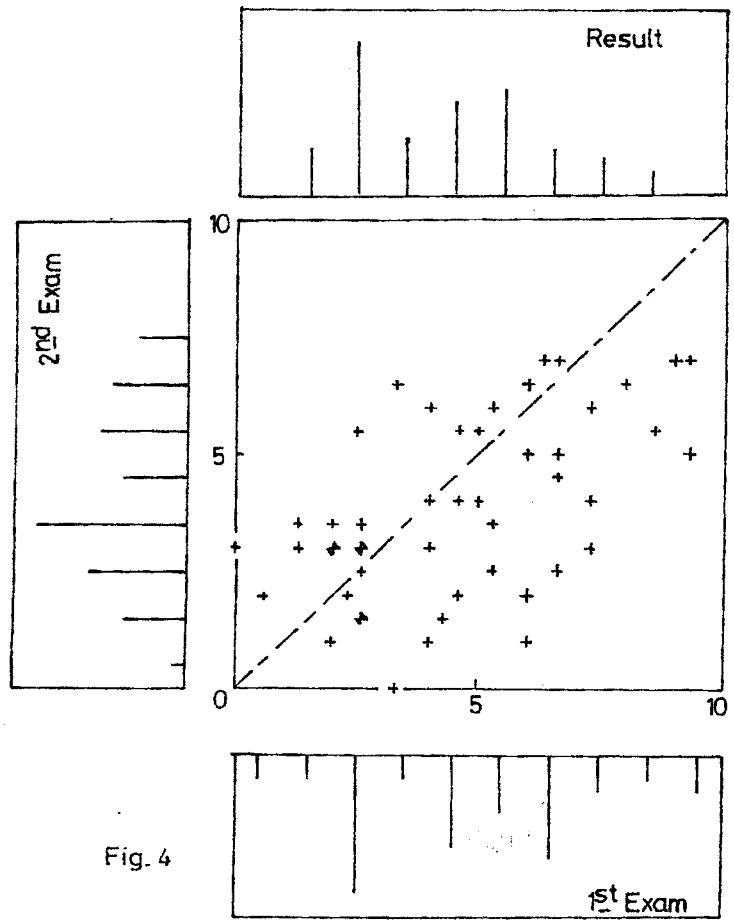

Fig. 4

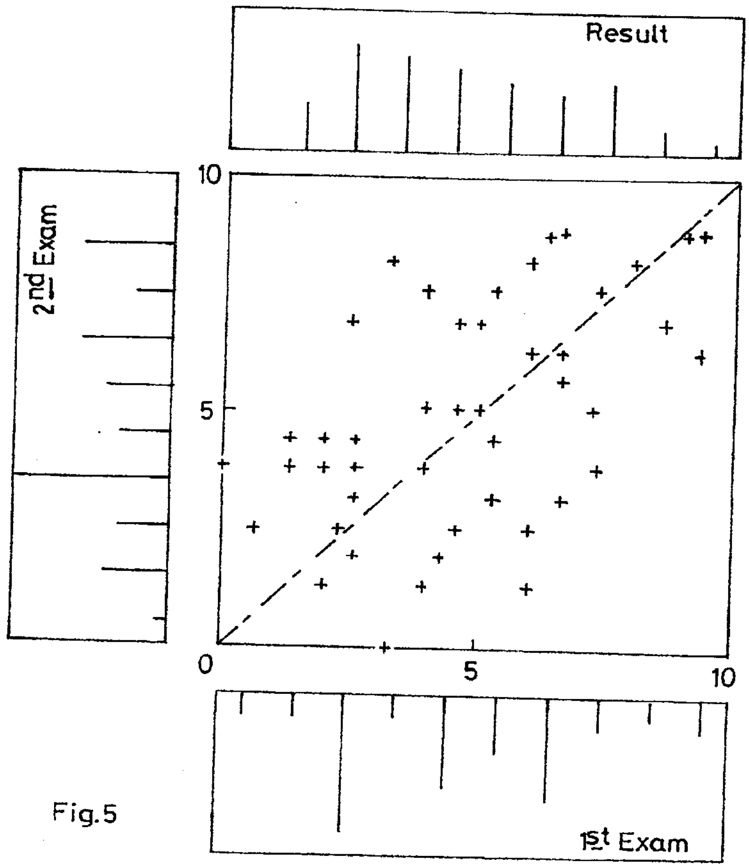

Fig.5